Locating RFID Tags

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C-LAB Report

Vol. 8 (2009) No. 3

Cooperative Computing & Communication Laboratory

ISSN 1619-7879

C-LAB ist eine Kooperation
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C-LAB Report

Herausgegeben von
Published by

Dr. Wolfgang Kern, Siemens AG
Prof. Dr. Franz-Josef Rammig, Universität Paderborn

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ISSN 1619-7879

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Abstract

Computing the position of an RFID tag based on signals received at several RFID readers can be done with a mathematical model. However, real world constraints impact this approach. In this paper we detail a combination of strict mathematics and heuristic approaches for a specific set of tags and readers.

Contents

1 Introduction

We want to determine the position of an RFID tag based on signals received at different RFID reader stations.

For our purposes, a very limited model of RFID technology is sufficient. This will be described in the next section.

1.1 RFID reader and tags

An RFID reader is an equipment producing electromagnetic fields in its vicinity. An RFID tag is an equipment able to manipulate this electromagnetic field. This manipulation can be detected by the reader. This way, information is transmitted between reader and tag.

1.2 General Approaches

Determing the position of an RFID tag attached to a moving object – e.g. a person – based on signals received by different RFID readers can be done in several different ways. One approach is to determine the signal strengths at different locations in the tracking area and create a map of this area showing the signal levels. When using this signal map, the real position is determined by comparing the actual signals with the ones from the map.

Another approach is to use a simple mathematical model based on balls and their intersections. In this paper we use this approach.

When there is no abstacle, the electromagnetic field is a ball with the reader as centre. All points with the same distance from the centre receive the same signal strength.

Model assumptions

We have the following assumptions for our model:

1. There is a set of RFID readers with fixed locations.
2. There are no obstacles: the signal strength of the electromagnetic field from a specific RFID is dependent on distance only.

This assumptions may sound naiive, and indeed they are. Nevertheless, this helps us in creating a basic model. At the end of this paper we discuss how to deal with obstacles.

2 Behaviour of RFID Reader

Each RFID reader is capable of generating a sequence of n different signal strengths $0 < s_0 < s_1 < \ldots < s_{i-i} < s_i \ldots < s_{n-1}$. In our special case we have $n = 4$ but this is not essential for the following considerations. In one round, the reader starts the first step by sending a signal of strength s_{n-1} and waits for reactions from tags for this signal strength. This is repeated with decreasing signal strengths until s_0 is reached.

We consider a fixed RFID tag with distance $r > 0$ from a fixed RFID reader. Let's assume the tag is receiving signals from the reader.

Let $s \in \{ s_{0,} s_{1,} \ldots , s_{i-i} , s_i , \ldots , s_{n-1} \}$ be the weakest signal that is received by the tag. Let r_i be the largest possible distance where signals of strength s_i can be received by the RFID tag.

When the tag receives signals of strength s_i it will receive stronger signals as well and thus is located within the ball of radius r_i . If it does not receive weaker signals $s < s_i$ it is located outside the ball of radius r_{i-1} . Together, we have the condition $r_{i-1} < r \leq r_i if\ s = s_i$.

Written in an algorithmic pseudo language, each reader executes the following steps:

```
Forever do
      let dist = 0                    // this means no signal received
      for signal strength si from s4 to s1 do
           send signal of strenght si
           wait for reaction from RFID tags nearby
           timeout according to predefined value …
           if reaction from tag
                  dist = di
           endif
      end for
      if dist > 0
           echo 'tag is within distance ' . dist . ' from reader'
      else
           echo 'reader does not see tag'
      endif
done
```

3 Geometrical model of RFID Reader Locations

For determing the position of one tag between different readers, we use the following approach:

Whenever there are distance values from at least 3 different readers available, we skip all but the 3 most tight circles. This results in 3 different circles.

We consider plain circles, but the computation is very similar in 3 dimensions as well. A detailed description of this concepts and the following mathematical results used can e.g. be found in [6].

Notation: In the following, capital letters always describe **vectors** and lowercase letters always describe **scalar** (or real) values. Vectors consist of components of real values, That means *M* is *(m1, m2)*.

Two consecutive capital letters denote a **scalar product**, i.e.

$MN = m1*n1 + m2*n2$ and $M^2 = MM$.

For each pair of circles *K1* and *K2* with centres *M* and *N, M ≠ N,* there is a straight line *g* running throught both centres. Let *r* be the radius of *K1* and *s* be the radius of *K2*. If *r + s* is larger than the distance between *M* and *N*, then there are 2 different intersection points *Q* and *T* between *K1* and *K2*.

Let *h* be the straight line connecting *Q* and *T*. Then, *h* is perpendicular on *g*.

We compute the point *P* which is just the intersection of the straight lines *g* and *h*.

For all points *X* on the circular line of *K1*, we have the following equation:

$$(1)\ (X-M)^2 - r^2 = 0$$

and correspondingly for *K2*:

$$(2)\ (X-N)^2 - s^2 = 0$$

For the straight line *g* we have

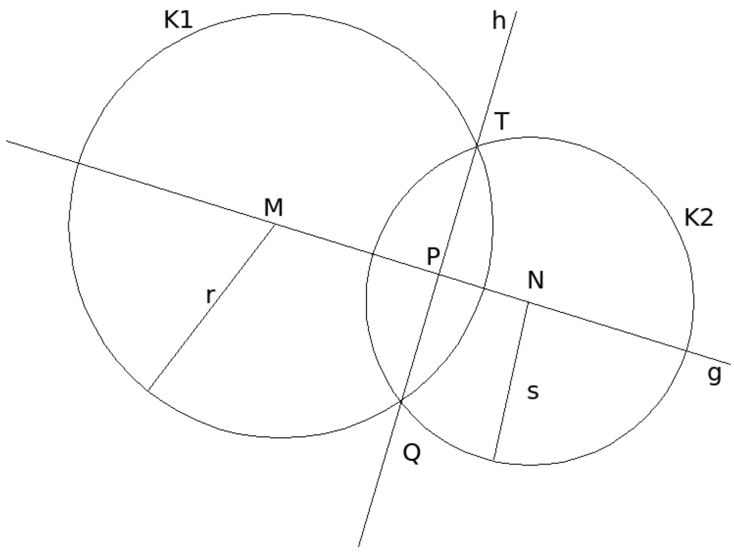
$$(3)\ X = M + t(N-M)$$

Subtracting (2) from (1) gives the following condition for *T* and *Q*:

$$(4)\ 0=(X-M)^2-r^2-((X-N)^2-s^2)=X^2-2\mathrm{XM}+M^2-r^2-(X^2-2\mathrm{XN}+N^2-s^2)= 2\mathrm{X}(N-M)+M^2-N^2-r^2+s^2$$

This is just the definition of the straight line *h*.

Since *P* must fullfil both (3) and (4), we can insert the value for *X* from (3) in (4) and continue as follows:



$$\begin{aligned}(4')\ 0 &= 2(M+t(N-M))(N-M)+M^2-N^2-r^2+s^2\\ &= 2MN-2\mathrm{M}^2+2t(N-M)^2+M^2-N^2-r^2+s^2\\ &= 2t(N-M)^2-(N-M)^2-r^2+s^2\end{aligned}$$

This can be solved to show *t:*

$$(5)\ t=\frac{(N-M)^2+r^2-s^2}{2(N-M)^2}$$

And thus *P* is according to (3)

$$(6)\ P=M+\frac{(N-M)^2+r^2-s^2}{2(N-M)^2}(N-M)$$

P is always a point on *g.* If *Q* and *T* coincide, which is true for

$$(N-M)^2=(r+s)^2$$

we have the situation where *h* is a tangent at *K1* and *K2* in *P*:

$$(6')\; P \;=\; M+\frac{(N-M)^2+r^2-s^2}{2(N-M)^2}(N-M) \;=\; M+\frac{(r+s)^2+(r-s)(r+s)}{2(r+s)^2}(N-M)$$
$$\;=\; M+\frac{2r(r+s)}{2(r+s)^2}(N-M) \;=\; M+\frac{r}{r+s}(N-M)$$

P is always a point on *g* but not necessarily between *M* and *N*. If

$r^2-s^2>(N-M)^2$ we have for the fraction in (6):

$$(6'')\; \frac{(N-M)^2+r^2-s^2}{2(N-M)^2}>1$$

Inserted as value for *t* in (3) this means we have the situation of the following figure, where *P* is not between *M* and *N.*

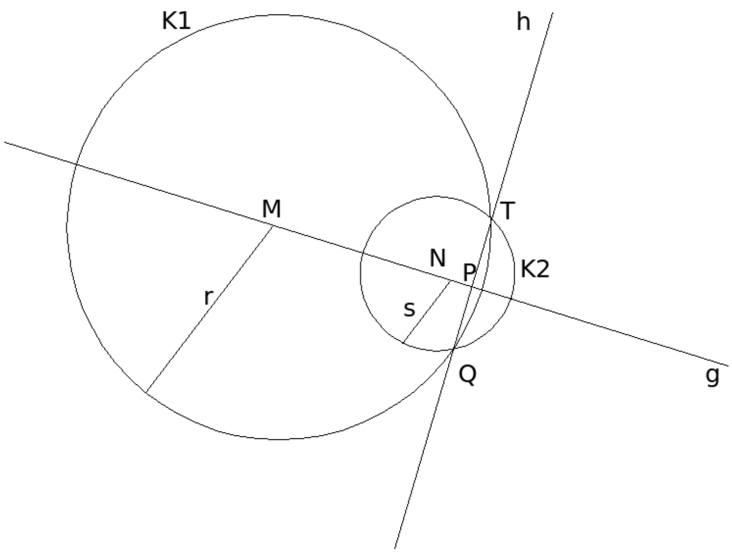
4 Geometrical conditions

In order to exclude pathological situations, we have the following requirements.

The circles do not collapse:

$$(R1)\;\; r>0$$

$$(R2)\;\; s>0$$



The circles have different centres:

$$(R3)\ \ M \neq N$$

The centre of *K2* is not contained in *K1*:

$$(R4)\ \ (N-M)^2 > r^2$$

or alternatively: circle *K2* is not completely inside of *K1 (*tangent is allowed):

$$(R4')\ \ (N-M)^2 \geq (r-s)^2$$

The centre of *K1* is not contained in *K2*:

$$(R5)\ \ (N-M)^2 > s^2$$

or alternatively: circle *K1* is not completely inside of *K2 (*tangent is allowed):

$$(R5')\ \ (N-M)^2 \geq (s-r)^2$$

And when all these conditions are fulfilled, there is an intersection point *P* when the following condition is met:

$$(R6)\ \ (N-M)^2 \leq (r+s)^2$$

In the case of equality *h* is tangent at both circles.

5 Position of RFID Tag

Having computed (6) for the 3 corners *A, B, C* of all 3 pairs of combinations from the 3 circles, the actual position of the tag can be guessed as the centre of gravity *S* of the corresponding triangle, which is just

$$(7)\ S = \frac{A+B+C}{3}$$

6 Adaptation of Position

Taking the centry of gravity can mislead for triangles where e.g. *A* and *C* are closed to each other but *B* is far away. In this case, the centre of gravity *S* is too far away from both *A* and *C* and too close to *B* in contrast to the expected position of tracking.

For the following discussions, let *E* point to the middle of the segment between *A* and *C*, i.e.

$$(8)\ E=\frac{A+C}{2}$$

We move *B* on the line connecting *B* and *E* towards *E,* up to a distance equal to the one between *A* and *C.* If *G* denotes the new position of *B*, we have a new triangle *(A, G, C)* where the formula (7) can be applied to determine the tracking position, of course with *G* substituting *B.*

Being on the line between *E* and *B* with the same distance as between A and C gives the following condition for *G:*

$$(9)\ G=E+\sqrt{\frac{(A-C)^2}{(B-E)^2}}(B-E)$$

Remember that there is a scalar product under the root, so we cannot shortcut here. Now (8) helps to substitute *E*, thus (9) becomes:

$$(10)\ G=\frac{A+C}{2}+\sqrt{\frac{(A-C)^2}{(\frac{2\mathrm{B}-A-C}{2})^2}}(\frac{2\mathrm{B}-A-C}{2})=\frac{A+C}{2}+\frac{\sqrt{(A-C)^2}}{\sqrt{(2\mathrm{B}-A-C)^2}}(2\mathrm{B}-A-C)$$

For the last equation we have applied the linearity of the scalar product. Now let us determine the terms under the root:

$$(11)\ (A-C)^2=(a_1-c_1)^2+(a_2-c_2)^2$$

and

$$\begin{aligned}(12)\ &(2\mathrm{B}-A-C)^2=(2\mathrm{b}_1-a_1-c_1)^2+(2\mathrm{b}_2-a_2-c_2)^2\\ &=4\mathrm{b}_1^2-2a_1b_1-2b_1c_1-2a_1b_1+a_1^2+a_1c_1-2b_1c_1+a_1c_1+c_1^2\\ &+4\mathrm{b}_2^2-2a_2b_2-2b_2c_2-2a_2b_2+a_2^2+a_2c_2-2b_2c_2+a_2c_2+c_2^2\\ &=a_1^2-4a_1b_1+2a_1b_1+4b_1^2-4b_1c_1+c_1^2\\ &+a_2^2-4a_2b_2+2a_2b_2+4b_2^2-4b_2c_2+c_2^2\end{aligned}$$

Using (11) and (12) show for the components of G directly from (10):

$$(13)\ g_1=\frac{a_1+c_1}{2}+\sqrt{\frac{(a_1-c_1)^2+(a_2-c_2)^2}{a_1^2-4a_1b_1+2a_1b_1+4b_1^2-4b_1c_1+c_1^2+a_2^2-4a_2b_2+2a_2b_2+4b_2^2-4b_2c_2+c_2^2}}(2b_1-a_1-c_1)$$

and the same for the second component:

$$(14)\ g_2=\frac{a_2+c_2}{2}+\sqrt{\frac{(a_1-c_1)^2+(a_2-c_2)^2}{a_1^2-4a_1b_1+2a_1b_1+4b_1^2-4b_1c_1+c_1^2+a_2^2-4a_2b_2+2a_2b_2+4b_2^2-4b_2c_2+c_2^2}}(2b_2-a_2-c_2)$$

For ease of computation, it might be better to use a product version, so (13) becomes (15):

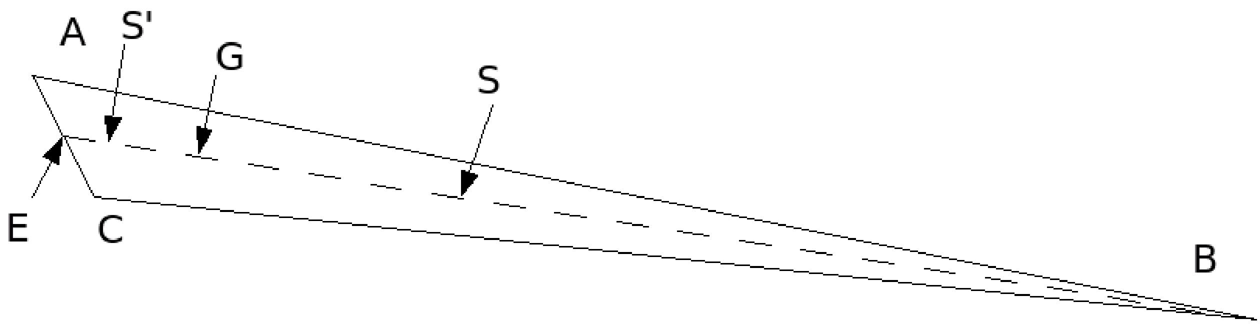
$$(15)\ g_1=\frac{a_1+c_1}{2}+\sqrt{\frac{(a_1-c_1)^2+(a_2-c_2)^2}{(2b_1-a_1-c_1)^2+(2b_2-a_2-c_2)^2}}(2b_1-a_1-c_1)$$

and 14 becomes (16):

$$(16)\ g_2=\frac{a_2+c_2}{2}+\sqrt{\frac{(a_1-c_1)^2+(a_2-c_2)^2}{(2b_1-a_1-c_1)^2+(2b_2-a_2-c_2)^2}}(2b_2-a_2-c_2)$$

6.1 Example

Let *A=(1,2), B=(21,6), C=(2,4)*, see the following picture where the y-coordinate runs downwards.



We do the following computations:

$$(17)\ S=\frac{A+B+C}{3}=(\frac{1+21+2}{3},\frac{2+6+4}{3})=(8,4)$$
$$E=\frac{A+C}{2}=(\frac{1+2}{2},\frac{2+4}{2})=(1.5,3)$$
$$2\mathrm{b}_1-a_1-c_1=2*21-1-2=39$$
$$2\mathrm{b}_2-a_2-c_2=2*6-2-4=6$$
$$g_1=\frac{1+2}{2}+\frac{\sqrt{1^2+2^2}}{\sqrt{39^2+6^2}}(39^2)\approx 3.71$$
$$g_2=\frac{2+4}{2}+\frac{\sqrt{1^2+2^2}}{\sqrt{39^2+6^2}}6\approx 3.34$$
$$S'\approx(\frac{1+2+3.71}{3},\frac{2+4+3.34}{3})\approx(2.237,3.113)$$

7 Determining RFID Reader Circle Radius Size

In this section we try to give a good estimation for the circle radius of a given RFID reader based on signal strength received from a fixed RFID tag.

The time is divided into cycles. In each cycle, a fixed tag sends for each of the 4 signal strengths several (4) packages containing the signal strength and the tag's id.

The reader receives a fraction of this packages and tries to determine the distance to the tag based on the signal strength and the fraction of packages received. This fraction can be adjusted to be the relative frequency of received packages compared to the total number of packages sent. This gives a bundle of functions computing the estimated radius for a given signal strength based on this frequency:

$$(18)\ f_{s_i}:[0,1]\rightarrow \mathbb{R} \quad for\ 1\leq i\leq 4$$

We assume the left and right border of the interval to contain useless values which we try to keep off from being used for the subsequent computations. The best values expected shall be described this way:

$$(19) f_{s_i}(x_1)=d_i \quad for\ 1\leq i\leq 5 \quad for\ example \quad x_1=0.5$$

Note that we have extended the values for the index i to contain an additional value (5). This will be needed later on.

The values computed here shall describe the radius for the fixed signal strength when receiving best.

This functions must be interconnected. The worst case values for smaller radius should correspond to the best cases values of the next larger radius:

$$(20) f_{s_i}(x_0)=f_{s_i+1}(x_1) \quad for\ 1\leq i\leq 4 \quad for\ example \quad x_0=0$$

Applying (19) to this gives:

$$(21) f_{s_i}(x_0)=d_{i+1} \quad for\ 1\leq i\leq 4$$

Assuming the functions are linear (which means straight lines) in between, we have the following solution:

$$(22) f_{s_i}(x) = \frac{d_i - d_{i+1}}{x_1 - x_0}(x - x_0) + d_{i+1} \quad for\, 1 \leq i \leq 4$$

And indeed, inserting the best value into (22) just gives:

$$f_{s_i}(x_1) = \frac{d_i - d_{i+1}}{x_1 - x_0}(x_1 - x_0) + d_{i+1} = d_i - d_{i+1} + d_{i+1} = d_i \quad for\, 1 \leq i \leq 4$$

But this is just (19). Furthermore:

$$f_{s_i}(x_0) = \frac{d_i - d_{i+1}}{x_1 - x_0}(x_0 - x_0) + d_{i+1} = d_{i+1} \quad for\, 1 \leq i \leq 4$$

Which is just (21).

8 Conclusions

Determining positions of RFID tags can start with a simple mathematical model but needs to be adjusted to real world constraints. Here, we took the freedom to introduce assumptions modifying our simple model towards real world effects. The realisation of this approach and the corresponding check within a real world project guided us and justified the appoach.

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